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A three-flavor AdS/QCD model with a back-reacted geometry

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Abstract: A fully back-reaction geometry model of AdS/QCD including the strange quark is described. We find that with the inclusion of the strange quark the impact on the metric is very small and the final predictions are changed only negligibly.

Keywords: QCD, AdS-CFT Correspondence.

Contents

1. Introduction

QCD [1] is considered to be a well-established theory for the strong interaction. In the high energy regime, we can use a perturbative approach to understand the theory. However, at low energy, because of the large coupling constant, perturbation theory is not applicable. In the low energy regime we can appeal to other methods of analysis, for instance chiral perturbation theory and lattice QCD.

Conjectured by Maldacena [2] in 1997, the AdS/CFT correspondence is a new approach to this difficult problem. This conjecture states that a string theory on $AdS_5 \times S^5$ is equivalent to a conformal theory on the boundary of AdS_5. QCD is classically but not quantum mechanically conformal. However, the AdS/CFT correspondence has provided important insights into QCD, such as confinement at large distances [3] and chiral symmetry breaking [4−11]. Currently these topics are very active areas of research.

The quantitative correspondence was specified in independent work by Gubser, Klebanov and Polyakov [12] and by Witten [13]

$$\left\langle e^{i\int d^4x\,\mathcal{O}\,\phi}\right\rangle_{\rm CFT} = \mathcal{Z}_{\rm SUGRA}\left(\phi(z)|_{z\to 0} = \phi\right) \tag{1.1}$$

which states that the generating functional for correlation functions with a source ϕ for some field theory operator is equivalent to the partition function of a supergravity theory where the boundary value of some supergravity field is the source for the field theory operator. The choice of supergravity field and field theory operator is a matter of matching the representations of the global symmetries of the two pair.

In recent years a new phenomenological approach, based on the rules of the AdS/CFT correspondence has been developed [6, 8]. This approach introduces a five-dimensional classical theory in an AdS_5 background where appropriate fields are included in the action to act as sources on the boundary for operators of a QCD-like theory. This original formulation included only light quark operators and gave a phenomenological model of chiral symmetry breaking. In the five-dimensional theory the symmetry is a gauge symmetry and a simple Higgs mechanism is set up to model chiral symmetry breaking in the four-dimensional theory.

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The results from these relatively simple and phenomenological models are remarkable and the simplest realization gives predictions for several meson masses and decay constants with an average of around 15% error.

Since the introduction of this phenomenological action, many advances have been made to model QCD more accurately. These include the introduction of linear confinement via an appropriately chosen scalar field in the five-dimensional theory [14], the inclusion of gluon condensate contributions to QCD quantities [15] and studies of heavy quark potentials [16–18].

In [19], we considered the impact of a classical scalar field back-reacting on the geometry. In this case the impact on the geometry was most strongly affected by the condensate of light quarks.

The strange quark was introduced [11] in order to study the kaon sector and found that reasonably accurate predictions could be found for these mesons, too.

In this paper we ask what the impact of the strange quark on the geometry will be. We may expect that as the chiral symmetry is broken more explicitly for the strange quark the effect of the strange quark condensate on the dynamics of the theory may be less pronounced.

2. Back reaction on the geometry

In this section, we consider the impact of one scalar field on the metric. The total field content is the gravitational field plus the scalar field, which will be responsible for chiral symmetry breaking. The Lagrangian is given by

$$S = \int d^5x \sqrt{g}(-R + Tr(\partial\phi)^2 + V(\phi)), \tag{2.1}$$

R is the five-dimensional Ricci scalar, and the metric is

$$ds^2 = e^{-2A(y)} dx_\mu dx^\mu - dy^2. \tag{2.2}$$

The Ricci scalar R is given by

$$R(y) = 20A'^2(y) - 8A''(y)\ . \tag{2.3}$$

From the action, one can find the equations of motion for the scalar field and for the metric tensor,

$$\frac{1}{2} g_{PQ}[-R + Tr(\partial_M\phi\partial^M\phi + V(\phi))] + R_{PQ} - Tr\partial_P\phi\partial_Q\phi = 0\,, \tag{2.4}$$

and

$$Tr\frac{\partial V(\phi)}{\partial\phi} = \frac{2}{\sqrt{g}} Tr\partial_P(\sqrt{g} g^{PQ}\partial_Q\phi)\,, \tag{2.5}$$

which gives

$$6A''(y) - 12A'^2(y) + Tr(V(\phi) - \phi'^2(y)) = 0 \tag{2.6}$$

$$12A'^2(y) - V(\phi) = Tr\phi'^2(y) \tag{2.7}$$

$$\phi''(y) - 4A'(y)\phi'(y) + \frac{1}{2}\frac{\partial V(\phi)}{\partial\phi} = 0. \tag{2.8}$$

Eq. (2.6) and eq. (2.7) give

$$3A''(y) = Tr\phi'^2(y) \tag{2.9}$$

and

$$3A''(y) - 12A'^2(y) + V(\phi) = 0\ . \tag{2.10}$$

From eq. (2.9), the function of $A(y)$ can be obtained, given a solution for ϕ. Then from eq. (2.10), one can find the potential $V(\phi)$. So, at no point do we need to rely on numerical techniques.

We now give an example and show how to find the warp factor in the metric function in the presence of a scalar field. It has been known that scalar field is the source of operator $\bar{q}q$ and the potential $V(\phi)$ includes the mass term $3\phi^2$. Considering the UV behavior $y \to -\infty$, $A(y) = y$, and keeping the lowest nontrivial mass term in the potential, we get from eq. (2.8) the following solution

$$\phi(y) = \frac{m_q}{2}e^y + \frac{\sigma}{2}e^{3y}\ , \tag{2.11}$$

here for later use, we suppose m_q and σ are 3 by 3 matrices

$$m_q = diag(m, m, m_s), \quad \sigma = diag(c, c, c_s).$$

Taking the scalar field in eq. (2.11) as the given solution, we shall be able to obtain the warp factor from eq. (2.9) and the Higgs potential from eq. (2.10).

It is not difficulty to check that

$$Tr\phi'^2(y) = 2\left(\frac{3}{2}ce^{3y} + \frac{1}{2}me^y\right)^2 + \left(\frac{3}{2}c_se^{3y} + \frac{1}{2}m_se^y\right)^2. \tag{2.12}$$

From eq. (2.9) and the UV boundary condition $A'(y)|_{y\to-\infty} = 1$, the warp factor $A(y)$ is found to be

$$A(y) = y + \frac{1}{8}\left(\frac{1}{3}c^2e^{6y} + \frac{1}{6}c_s^2e^{6y} + \frac{1}{2}c\ m\ e^{4y} + \frac{1}{4}c_s\ m_s\ e^{4y}\right). \tag{2.13}$$

It is seen that the UV behavior of the metric is not greatly modified by the back reaction of this scalar field. The potential $V(\phi)$ in eq. (2.1) deserves a comment. From eq. (2.10), with $A(y)$ in eq. (2.13), one can calculate $V(\phi)$. In general, $V(\phi)$ will get corrections due to interactions with the metric field. This example demonstrates that the system is self-consistent.

3. A phenomenological model

In such a model of QCD, the three relevant operators are $\bar{q}_L^\alpha q_R^\beta$, $\bar{q}_{L,R}\gamma^\mu t^a q_{L,R}$.

Now, let's consider the following action

$$S = \int d^5x\sqrt{g}\left\{-R + Tr\left(|D\phi|^2 + V(\phi) - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right)\right\} \tag{3.1}$$

where $D_M\phi = \partial_M\phi - iA_{LM}\phi + i\phi A_{RM}, A_{L,R} = A^a_{L,R}t^a$ and $F_{MN} = \partial_M A_N - \partial_N A_M - i[A_M, A_N]$, where $Tr[t^a, t^b] = \frac{1}{2}\delta^{ab}$. We define the vector and axial-vector gauge bosons to be $V_M = \frac{1}{2}(A_{LM} + A_{RM})$ and $A_M = \frac{1}{2}(A_{LM} - A_{RM})$ respectively. Following ref. [6], we choose the gauge $V_z = A_z = 0$. From this action, the expectation value of the scalar field is found to be that chosen in eq. 2.11, m_q is corresponding to the quark mass matrix, σ is related to the quark condensate: $\sigma^{\alpha\beta}{=}\langle\ \bar{q}^\alpha q^\beta\rangle$, which can be shown by matching the four-dimension effective Lagrangian to chiral Lagrangian [8]. Substituting $\phi = <\phi> e^{i2t^a\pi^a(x,y)}$ back into the action, the mass matrix of V_M and A_M bosons are found to have the following forms [11].

$$M_V^2 = \begin{pmatrix} \mathbf{0_{3\times 3}} & 0 & 0 \\ 0 & \frac{1}{4}\left(\hat{m} - \hat{m}_s\right)^2 z^2 \mathbf{1_{4\times 4}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{3.2}$$

and

$$M_A^2 = \begin{pmatrix} \hat{m}^2 z^2 \mathbf{1_{3\times 3}} & 0 & 0 \\ 0 & \frac{1}{4}\left(\hat{m} + \hat{m}_s\right)^2 z^2 \mathbf{1_{4\times 4}} & 0 \\ 0 & 0 & \frac{1}{3}\left((\hat{m})^2 + 2\left(\hat{m}_s\right)^2\right) z^2 \end{pmatrix}, \tag{3.3}$$

where $\hat{m} = m + cz^2$ and $\hat{m}_s = m_s + c_s z^2$. For convenience we have made change of variable $z = e^y$. Denote $\tilde{V}^a_{M\perp}(q,z)$ and $\tilde{A}^a_{M\perp}(q,z)$ as the normalizable modes of four-dimensional Fourier transformed transverse field $V^a_{M\perp}(x,z)$ and $A^a_{M\perp}(x,z)$ respectively, they satisfy the following equations of motion (no summation)

$$\left[\partial_z^2 + \partial_z\left(\ln\tilde{A}(z)\right)\partial_z + \left(q^2 - (g_5^2\tilde{A}^2(z)M_V^2)_{aa}\right)\right]\tilde{V}^a_{M\perp}(q,z) = 0, \tag{3.4}$$

and

$$\left[\partial_z^2 + \partial_z\left(\ln\tilde{A}(z)\right)\partial_z + \left(q^2 - (g_5^2\tilde{A}^2(z)M_A^2)_{aa}\right)\right]\tilde{A}^a_{M\perp}(q,z) = 0, \tag{3.5}$$

where

$$\tilde{A}(z) = \frac{1}{z}exp\left(-\frac{1}{8}\left(\frac{1}{3}c^2z^6 + \frac{1}{6}c_s^2z^6 + \frac{1}{2}cz^4m + \frac{1}{4}c_sz^4m_s\right)\right),$$

with boundary conditions

$$\partial_z\tilde{V}^a_{M\perp}(q,z)|_{z=z_{\rm IR}} = 0, \qquad \tilde{V}^a_{M\perp}(q,z)|_{z=\epsilon} = 0,$$

similarly for $\tilde{A}^a_{M\perp}$. The mass of the vector and axial vector mesons can be obtained by solving the eigenvalue equations eq. (3.4) and eq. (3.5) with $q^2 = m_V^2$ and $q^2 = m_A^2$, respectively. The decay constants of these mesons can be obtained from an arbitrary component of vector field via [6]

$$F_{V^a} = \frac{1}{g_5}\frac{|\partial_z^2\tilde{V}^a_\perp(M_{V^a},z)|}{N_{V^a}}|_{z=0}, \tag{3.6}$$

where N_{V^a} is the normalization factor defined by an arbitrary component of vector field

$$N_{V^a} = \int_0^{z_{\rm IR}} dz\,\tilde{A}(z)\,|\tilde{V}^a_\perp(M_{V^a},z)|^2,$$

$z_{\rm IR}^{-1}$	m	$c^{\frac{1}{3}}$	m_s	$c_s^{\frac{1}{3}}$
320.55	2.28	328.5	138.5	176

Table 1: Fit results for the free parameters in units of MeV.

similarly for F_{A^a}. The mass of the pseudoscalar mesons can be obtained by solving the following equations (no summation)

$$\begin{aligned}
&\left(\partial_z^2+\partial_z\left(\ln\tilde{A}(z)\right)\partial_z\right)\phi^a+g_5^2\tilde{A}^2(z)\left(M_A^2\right)_{aa}(\pi^a-\phi^a)=0, \qquad (3.7)\\
&\partial_z\left(\tilde{A}^3(z)\left(M_V^2+M_A^2\right)_{aa}\partial_z\pi^a\right)=q^2\tilde{A}^3(z)\Bigg(\left(M_V^2+M_A^2\right)_{aa}\left(\frac{1}{2}\phi^a-\pi^a\right)\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad+\left(M_A^2-M_V^2\right)_{aa}\frac{1}{2}\phi^a\Bigg),
\end{aligned}$$

with boundary conditions

$$\partial_z\phi^a(z=z_{\rm IR})=\partial_z\pi^a(z=z_{\rm IR})=\phi^a(z=0)=\pi^a(z=0)=0$$

where ϕ^a is defined as the longitudinal part of A_μ^a, i.e., $\partial_\mu\phi^a=A^a_{\mu||}$.

The decay constants of the pseudoscalar is calculated from an arbitrary component of axial vector field via [6]

$$f_{P^a}=-\frac{1}{g_5^2}\frac{\partial_z\tilde{A}^a_\perp(0,z)}{z}|_{z\to 0}, \qquad (3.8)$$

with $\tilde{A}^a_\perp(0,z)$ are given by the solution of eq. (3.5) with $q^2=0$ and boundary conditions

$$\partial_z\tilde{A}^a_\perp(0,z)|_{z=z_{\rm IR}}=0, \qquad \tilde{A}^a_\perp(0,z)|_{z=0}=1.$$

The IR cutoff $z_{\rm IR}$ is introduced to generate a mass gap. The fifth dimension is taken as an interval from 0 to $z_{\rm IR}$.

The model now has six free parameters: $g_5^2,\ z_{\rm IR},\ m,\ c,\ m_s,\ c_s$. Among them g_5^2 can be obtained by comparing the vector-vector two point function obtained from the OPE in large N_c limit [20] to that obtained by using the holographic recipe [6], which leads to the value $g_5^2=\frac{N_c}{12\pi^2}$. Thus there are actually five free parameters left, we shall use an iterative method to fit the five free parameters.

Let us begin with fitting the parameters without the back reaction. Namely, as a starting point we choose $\tilde{A}(z)=\frac{1}{z}$. Using the following experimental data: $m_\pi=139.6\,$MeV, $f_\pi=92.4\,$MeV, $m_\rho=775.8\,$MeV, $m_{K1A}=1339\,$MeV, and a semi-global fit for m_{K^*}, we can then use an iterative method to fix the free parameters in order to minimize the rms error on the remaining data. The final fit results are shown in table 1. Having fixed the free parameters, we can calculate the remaining mesons masses and decay constants. In table 2 and table 3, we show the mass and decay constants of vector mesons and axial vector mesons respectively.

observation	$value(MeV)(\%error)$
m_π	139.6*
f_π	92.4*
m_{a1}	1364(10.9)
$\sqrt{F_{a1}}$	440(1.6)
$m_{K_{1A}}$	1339*
$\sqrt{F_{K_{1A}}}$	$435(\sqrt{F_{K_1}(1400)} \sim 454^\dagger)$
m_{A3}	1344
$\sqrt{F_{K_{A_3}}}$	412

Table 2: Axial vector meson results calculated with a back reacted geometry and the free parameters given in table 1. Experimental values are chosen as the midpoint of those in [21]. The decay constant of the a_1 is compared with the lattice result [22]. The values with $*$ indicates that this value is used to fix the free parameters, all other values are predictions. Numbers in brackets give the percentage error. The value with † is taken from [23].The axial vector meson A3 corresponds to the isosinglet meson in the octet.

observation	$value(MeV)(\%error)$
m_ρ	775.8*
$\sqrt{F_\rho}$	348.8(1.1)
$m_{\rho'}$	1781
$\sqrt{F_{\rho'}}$	658
m_{K^*}	812*(9)
$\sqrt{F_{K^*}}$	$328(11^\dagger)$
m_{V_3}	m_ρ
$\sqrt{F_{V_3}}$	$\sqrt{F_\rho}$

Table 3: Vector meson results calculated with a back reacted geometry and the free parameters given in table 1. The ρ' is the first excited state of the ρ meson. Experimental values are chosen as the midpoint of those in [21]. The values with $*$ indicates that this value is used to fix the free parameters, all other values are predictions. Numbers in brackets give the percentage error. The value with † is taken from lattice predictions [24].The vector meson V3 corresponds to the isosinglet meson in the octet.

Having fitted the free parameters and calculated the remaining meson properties we can also calculate the Ricci scalar for the back reacted geometry:

$$R(z) = -12c^2z^6 - 6c_s^2z^6 - 8cz^4m - 4c_sz^4m_s + \frac{5}{16}\left(8 + 2c^2z^6 + c_s^2z^6 + 2cz^4m + c_sz^4m_s\right)^2. \quad (3.9)$$

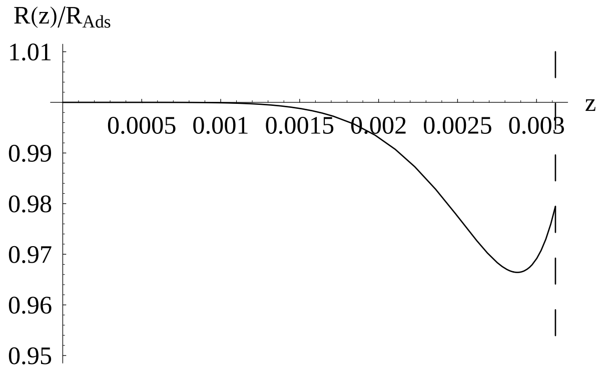

Figure 1: The variation of the Ricci scalar as a function of radial distance in our model with with parameters in table 1.

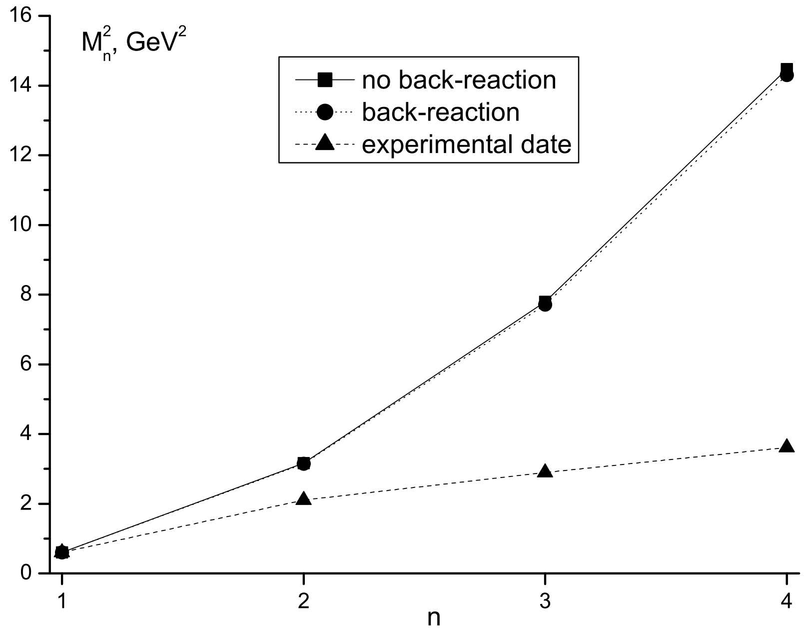

Figure 2: Experimental values for $M^2_{\rho^*}(GeV^2)$ against theoretical values both with and without back-reaction. The results without back-reaction are calculated with $z_{\rm IR}^{-1} = 322.6MeV$ and those with back-reaction are calculated using the parameters given in table 1.

We plot the curvature as a function of the radial distance in the AdS space, see figure 1. Figure 1 shows that the back reaction has only a small impact on the scalar curvature in the interval $(0, z_{\rm IR})$ with a maximum of around 3% departure from the pure AdS result. For $z > z_{\rm IR}$ the impact is larger but has no effect on our results.

We also calculate the mass of ρ resonances which is shown in figure 2. However, because of the small difference between no-back-reaction case and back-reaction case, the two lines are almost indistinguishable on this scale. As the stringy effects are neglected in our present analysis, they are expected to become important in the UV. Thus, the reliability of the current models will diminish above the scale of chiral symmetry breaking (around $\Lambda_\chi \sim 4\pi f_\pi \sim 1.2\,\mathrm{GeV}$).

The main conclusion of this calculation is that even with the addition of strange quark dynamics the geometry and hence the spectra of masses and decay constants are not heavily affected. This is a non-trivial statement about the impact of the strange quark on chiral dynamics.

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References

[1] H. Fritzsch, M. Gell-Mann and H. Leutwyler, *Advantages of the color octet gluon picture*, *Phys. Lett.* **B 47** (1973) 365.

[2] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [hep-th/9711200].

[3] J. Polchinski and M.J. Strassler, *Hard scattering and gauge/string duality*, *Phys. Rev. Lett.* **88** (2002) 031601 [hep-th/0109174].

[4] J. Babington, J. Erdmenger, N.J. Evans, Z. Guralnik and I. Kirsch, *Chiral symmetry breaking and pions in non-supersymmetric gauge/gravity duals*, *Phys. Rev.* **D 69** (2004) 066007 [hep-th/0306018].

[5] M. Kruczenski, D. Mateos, R.C. Myers and D.J. Winters, *Meson spectroscopy in AdS/CFT with flavour*, *JHEP* **07** (2003) 049 [hep-th/0304032].

[6] J. Erlich, E. Katz, D.T. Son and M.A. Stephanov, *QCD and a holographic model of hadrons*, *Phys. Rev. Lett.* **95** (2005) 261602 [hep-ph/0501128].

[7] S. Hong, S. Yoon and M.J. Strassler, *On the couplings of vector mesons in AdS/QCD*, *JHEP* **04** (2006) 003 [hep-th/0409118].

[8] L. Da Rold and A. Pomarol, *Chiral symmetry breaking from five dimensional spaces*, *Nucl. Phys.* **B 721** (2005) 79 [hep-ph/0501218].

[9] J. Hirn, N. Rius and V. Sanz, *Geometric approach to condensates in holographic QCD*, *Phys. Rev.* **D 73** (2006) 085005 [hep-ph/0512240].

[10] K. Ghoroku, N. Maru, M. Tachibana and M. Yahiro, *Holographic model for hadrons in deformed AdS_5 background*, *Phys. Lett.* **B 633** (2006) 602 [hep-ph/0510334].

[11] J.P. Shock and F. Wu, *Three flavour QCD from the holographic principle*, *JHEP* **08** (2006) 023 [hep-ph/0603142].

[12] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[13] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[14] A. Karch, E. Katz, D.T. Son and M.A. Stephanov, *Linear confinement and AdS/QCD*, *Phys. Rev.* **D 74** (2006) 015005 [hep-ph/0602229].

[15] C. Csáki and M. Reece, *Toward a systematic holographic QCD: a braneless approach*, *JHEP* **05** (2007) 062 [hep-ph/0608266].

[16] H. Boschi-Filho, N.R.F. Braga and C.N. Ferreira, *Static strings in Randall-Sundrum scenarios and the quark anti-quark potential*, *Phys. Rev.* **D 73** (2006) 106006 [*Erratum ibid.* **74** (2006) 089903] [hep-th/0512295].

[17] H. Boschi-Filho, N.R.F. Braga and C.N. Ferreira, *Heavy quark potential at finite temperature from gauge/string duality*, *Phys. Rev.* **D 74** (2006) 086001 [hep-th/0607038].

[18] C.D. White, *The Cornell potential from general geometries in AdS/QCD*, *Phys. Lett.* **B 652** (2007) 79 [hep-ph/0701157].

[19] J.P. Shock, F. Wu, Y.L. Wu and Z.F. Xie, *AdS/QCD phenomenological models from a back-reacted geometry*, *JHEP* **03** (2007) 064 [hep-ph/0611227].

[20] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *QCD and resonance physics. Sum rules*, *Nucl. Phys.* **B 147** (1979) 385.

[21] Particle Data Group collaboration, S. Eidelman et al., *Review of particle physics*, *Phys. Lett.* **B 592** (2004) 1.

[22] N. Isgur, C. Morningstar and C. Reader, *The A1 in τ decay*, *Phys. Rev.* **D 39** (1989) 1357.

[23] M. Wingate, T.A. DeGrand, S. Collins and U.M. Heller, *Properties of the A1 meson from lattice QCD*, *Phys. Rev. Lett.* **74** (1995) 4596 [hep-ph/9502274].

[24] C.R. Allton, V. Gimenez, L. Giusti and F. Rapuano, *Light quenched hadron spectrum and decay constants on different lattices*, *Nucl. Phys.* **B 489** (1997) 427 [hep-lat/9611021].